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Research Article

Estimation of Reliability Function of Pepsi Cola Baghdad Machines Under Second Type Monitoring Data According to Rayleigh Distribution

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Abstract: This research aims to estimate the reliability function of the Rayleigh distribution Under Type-II-Censored Data, which represents. The time of the machine's operation until the failure, and three methods were chosen to estimate the reliability function, which are, the maximum likelihood method, the momentary method, and the least squares method. As the values of the reliability function were calculated for real data related to the machines of theof the machines of Baghdad Pepsi Cola Company, and the comparison was made between the methods of estimating reliability by choosing the largest arithmetic mean for the reliability of the machines, and the greatest possibility appeared the best method, and the researcher concluded that the average times working until holidays is (4) months with a reliability of 87%.

Keywords: Maximum Likelihood Estimation; Method of Moments; Rayleigh Distribution; Reliability Function; Type-II Censoring.

1. Introduction

As a result of the rapid development in technology and the wide spread of industry and its use in various fields of life, such as medicine, space research, communications .... and other other fields, and its many electronic, mechanical and electrical complications, the interest in reliability came and its idea appeared at the end of the forties with the beginning of World War II due to the increase in complex military equipment and as an attempt to solve a problem Failure in equipment, and the importance of reliability comes through several features, including predicting the total count of working and non-operating machines at any time, which enables decision-makers to develop future plans to manage work correctly and positively and avoid problems at work, and then this leads to achieving a good reputation For any factory or company, increase sales and achieve more profits. It is necessary to present some of the literature that addressed the research topic. In 2011, the researchers Makhoul and Ghanem [9] published a study in which they estimated the parameters of the two-parameter Weibull distribution according to White's method and then estimated the reliability function. The study was applied to data on maximum temperatures and rainfall for the city of Damascus for the time period (1988-2007). In the year (2013), Ismail K and others [8] published a study that deals with presenting modified methods for estimating the parameters of the two-parameter Weibull distribution under second-type observational data. Maximum likelihood estimators were derived and the EM algorithm was adopted in the estimation. Also in (2013), the researchers (Hong & Chien [7] published a study that included the adoption of second-type observational data (truncated data), especially when observations are difficult and expensive. Therefore, the principle of observational data was adopted in the estimation, and a comparison was made between the maximum likelihood method and other methods with different sample sizes and the use of the simulation method in the estimation. In (2018), researchers Al-Taie and Osama [6], presented a study that included estimating the reliability of the machines of the Sufiya factory for type 2 monitoring data, following the Weibull distribution, and the distribution parameters were estimated using the maximum probability

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and moments method, using the arithmetic mean as a comparison measure, and the preference of the maximum probability appeared and that the reliability rate on the machines is (87%) for every four days.

2. Research Problem

Often a lot of machines (devices) of different types are subject to stopping and failures, as a result of sudden and backup holidays, which affects the reliability of those machines (devices), and it is necessary to accurately estimate the reliability, in order to estimate the average and variance of the operating time until failure because they are very good indicators of follow-up Business in workshops and factories.

2.1. Research Purpose

The research aims to estimate the reliability function of monitoring data of the second type, subject to Rayleigh distribution for the machines of Baghdad Pepsi Cola Company underwhich contributes to providing good predictions for the possible failure times of the machines.

2.2 Reliability Function

It is the probability that the machine (the device) continues to operate successfully (without failure) during a certain period of time (0,t), meaning that the machine remains effective after the passage of time t, (0 t >) and its mathematical expression is:

$$R(t) = pr\ (T > t\ ) \quad \dots .(1)$$

t: The working time is greater or equal to zero.

T: The accumulated life of a given device during the period (0,t).

The formula for the dependency function in the continuous distribution is:

$$R(t) = \int_{t}^{Maxt} f(u)du \dots (2)$$

And the value of the reliability function at zero time is 1 and its value begins to decrease monotonously and its value becomes at the maximum time (Maxt) for the life of the machine equal to zero:

$$R(t = 0) = 1$$

$$\vdots$$

$$\vdots$$

$$R(t = Max) = 0$$

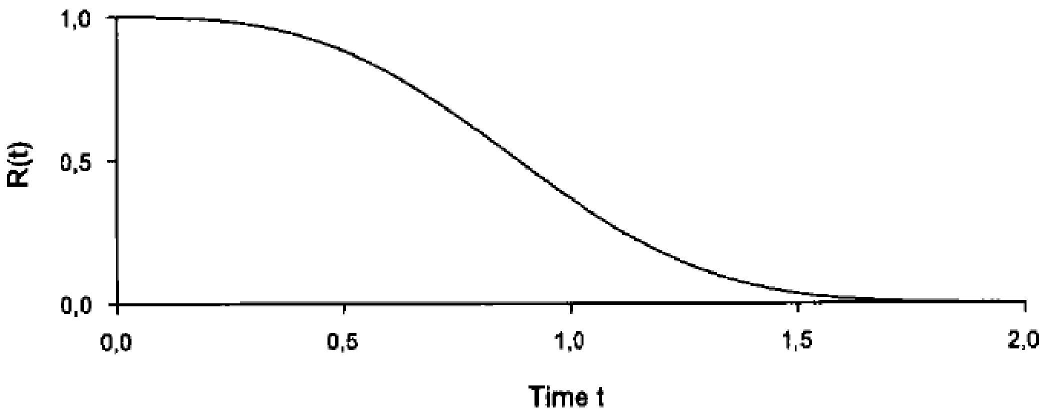

Figure 1. represents the reliability function

2.3 Type – II-Censored Data

It is one of the types of monitoring data, the researcher determines a number of sample units that are monitored until failure (r) fixed units. Therefore, the time of these units (t_r) is a random variable that we can determine after obtaining r of the failed units that were previously determined, and the number of remaining units is (n-r). The maximum likelihood function for this type of data is:

$$L = \frac{n!}{(n-r)!}\prod_{i=1}^{m} f(t,\theta)[R(t_r)]^{n-r} \quad \dots (3)$$

As:

$0 < t_1 < t_2 <\ldots< t_r$

n: Sample volume

r The number of units specified for failure

(n-r): The number of non-failed units after the test was stopped when the unit failed r

t_i: Time to work until failure

t_r: Time to operate the last unit r until failure

2.2 Rayleigh Distrbution

Rayleigh distribution is a special case of the two-parameter Weibull distribution when the shape parameter is equal to 2, and it is one of the important distributions in the survival analysis of many phenomena, and it is also a model of failure models. Rayleigh distribution was discovered by the English physicist (Lord Rayleigh). Another of its uses is to know the strength of wireless and radio signals during peak hours in communications. The probability density function p.d.f can be written as follows:

$$f(t:\theta) = \frac{1}{\theta^2}\, t_i\, e^{-t^2/2\theta^2} \quad \ldots(4)$$

$\theta : scal\ parameter\ \ , \theta > 0\ , t > 0$

As for the cumulative distribution and reliability functions, they are written in the following form:

$$F(t) = 1 - e^{-t^2/2\theta^2} \quad \dots (5)$$

$$R(t) = e^{-t^2/2\theta^2} \quad \dots (6)$$

3. Estimation Methods

There are many methods for estimating the parameters and reliability of the Rayleigh distribution, and some of these methods will be clarified in the case of type-II-censored data, and these methods include:

3.1. Maximum Likelihood Method (MLE)

This method is one of the important and commonly used estimation methods in estimation, and the reason for this is that the greatest possibility method possesses a set of good properties, including sufficiency and consistency at times, and it is more accurate than other methods, especially when the sample size increases as it is unbiased when the sample size is Great, and the goal of this method is to find estimated values for the parameters that we want to estimate by making the maximum possibility function for the random variables as large as possible, and denoting the greatest possibility function by the symbol (L).

If the random variable (T) has a probability density function, and as in equation (1), the maximum possibility function for the independent random variables$T_1, T_2, \dots \dots . T_n$ is:

$$\boldsymbol{L(T_1, T_2, \dots, T_n, \theta) = f\left(t_1, \theta\right). f(t_2, \theta) \dots f(t_n, \theta)}$$

$$\therefore L = \prod_{i=1}^{n} f(t_i\,, \theta)$$

Since the maximum possibility function for control data of the second type is:

$$L = \frac{n!}{(n-r)!}\prod_{i=1}^{r} f(t_i\,, \theta)\ \ [1 - F(t_r)]^{n-r}$$

By substituting the probability density function of the Rayleigh distribution into the maximum possibility function, we get the following equation:

$$L = \frac{n!}{(n-r)!}\prod_{i=1}^{r} \frac{1}{\theta^2}\, t_i\, e^{-t^2/2\theta^2}\ \ [R(t_r)]^{n-r}. \dots (7)$$

Substituting in the reliability function and assuming k=n!/(n-r)! We get the following:

$$= K\left[(\frac{1}{\theta^2})^r \prod_{i=1}^{r} t_i \quad e^{\frac{-\sum_{i=1}^{r} t_i^2}{2\theta^2}}\right]\left[e^{-t_r^2/2\theta^2}\right]^{n-r}$$

$$L = K(\frac{1}{\theta^2})^r \prod_{i=1}^{r} t_i \, e^{\frac{-\sum_{i=1}^{r} t_i^2}{2\theta^2}} \quad \left[e^{-t_r^2/2\theta^2}\right]^{n-r}$$

$$L = K(\frac{1}{\theta^2})^r \prod_{i=1}^{r} t_i \, e^{\frac{-1}{2\theta^2}} \quad \left[\sum_{i=1}^{r} t_i^2 + (\mathrm{n}-\mathrm{r})t_r^2\right] \quad \dots(8)$$

Taking the natural logarithm of both sides of the equation, we get:

$$LogL = logK + 2rLog\theta + \sum_{i=1}^{r} Logt_i \; - \frac{1}{2\theta^2}\left[\sum_{i=1}^{r} t_i^2 + (n-r)t_r^{\beta}\right] \dots(9)$$

$$\frac{\partial LogL}{\partial \theta} = \frac{-2r}{\theta} + \frac{1}{\theta^3}\left[\sum_{i=1}^{r} t_i^2 + (n-r)t_r^2\right] \dots(10)$$

$$\frac{\partial LogL}{\partial \theta} = 0$$

$$\hat{\theta}_{MLE} = \sqrt[2]{\frac{\sum_{i=1}^{r} t_i^2 + (n-r)t_r^2}{4r}} \dots(11)$$

3.2. Method of Moments (MOM)

This method is characterized by its ease, and therefore it is one of the common methods used in the field of parameter estimation.

Since the first visible moment of the Rayleigh distribution is defined as:

$$M_1 = E(t^1) = \theta\sqrt{\frac{\pi}{2}}$$

Since the first sampling moment is:

$$m_1 = \frac{\sum_{i=1}^{r} t_i}{r}$$

From the equation below, we get the estimate $\hat{\theta}$

$$M_1 = m_1$$

$$\hat{\theta} = \frac{\bar{t}}{1.253} \quad \dots(12)$$

3.3. Least Square Method (OLS)

The method of least squares depends on finding the estimators of shape and measure (B, θ), which are characterized by providing the smallest sum of squares error that we can find from the sum of the difference between the CDF function and one of the nonparametric estimators, as there are many nonparametric estimators and we will choose from them:

$$* \; \hat{F}_{1\,(x_i)} = \left(\frac{i}{n}\right)$$

$$* \; \hat{F}_{2(x_i)} = \left(\frac{i-0.5}{n}\right)$$

$$* \; \hat{F}_3(x_i) = \left(\frac{i}{n+1}\right)$$

as

i : represents the sequence of observations (ti) arranged in ascending order.

F^: represents the nonparametric expression of the function C.d.f

The mathematical formula adopted for the estimation is:

$$T_1 = \sum_{i=1}^{r} \left[\left\{ 1 - e^{-t^2/2\theta^2} \right\} - \left(\frac{i}{n} \right) \right]^2$$

$$T_2 = \sum_{i=1}^{r} \left[\left\{ 1 - e^{-t^2/2\theta^2} \right\} - \left(\frac{i - 0.5}{n} \right) \right]^2$$

$$T_3 = \sum_{i=1}^{r} \left[\{ 1 - e^{-t^2/2\theta^2} \} - \left(\frac{i}{n + 1} \right) \right]^2$$

as

Ti: the sum of the squares of the error.

i=1,2,3

An iterative method will be applied by giving the initial values of its parameter (θ= 1, 1.5 , 2), and using, ($\hat{\theta}$), for which the value T1,T2,T3 is as small as possible.

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4. Results and Discussion

4.1. SUM Data

The number of Pepsi Cola Baghdad machines was 33 machines, and the number of broken units was determined (r=10), at which point monitoring stops, i.e. when the tenth machine breaks down, data recording stops, and the number of remaining machines (n-r =22)10 times the machine worked were recorded until the breakdown occurred in 2023, and the number of machines was 3 3 machines, and when the ten machines broke down, data recording stopped.

4.2. TEST Data

To find out whether the data follow a Riley distribution, a goodness of fit test was used The value of (P-Value = 0.2334) appeared, which is greater than the level of significance (0.05), so we accept the null hypothesis, meaning that the data are distributed in a Rayleigh distribution.

4.3. Reliability Function Calculation

Table 1. Reliability.

	t_i	$\hat{R}_{ML}$	$\hat{R}_{OLS}$	$\hat{R}_{MO}$
	41	0.99	0.92	0.96
	50	0.93	0.91	0.95
	77	0.89	0.88	0.91
	79	0.86	0.86	0.83
	86	0.81	0.83	0.82
	98	0.79	0.81	0.77
	100	0.78	0.77	0.73
	112	0.69	0.71	0.68
	142	0.58	0.65	0.64
Standard deviation		.12460	.11533	.09139
mean	87.2222			.8156

Looking at the numbered table (1), we notice that the standard deviation of the reliability function values using the moment method (MO) is the smallest and therefore it is the best way to estimate the reliability function, and the average working time until the holidays is (87) days, which means that the factory machines can be relied upon by 81% for every 87 days.

5. Conclusions

The most effective method for calculating reliability was found to be the maximum likelihood function. It was observed that the reliability function values consistently decrease as operating time increases, remaining within the range of 0 to 1. Additionally, the analysis showed that the machines in the laboratory maintained a reliability level of 81% over a period of 87 days.

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