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Research Article

The Effect of the Discovery Learning Model with the Aptitude Treatment Interaction Strategy on the Ability to Understand Mathematical Concepts and Self-Efficacy of Class X Students of SMA 3 Tarakan

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Abstract: This study aims to: (1) assess the impact of the discovery learning model with the aptitude-treatment interaction (ATI) strategy on the mathematical concept comprehension and self-efficacy of 10th-grade students at SMA Negeri 3 Tarakan; (2) evaluate its effect on mathematical concept comprehension; and (3) determine its effect on self-efficacy. The study involved 10th-grade students from SMA Negeri 3 Tarakan in the 2024/2025 academic year. Two randomly selected classes were chosen as the sample: one experimental class using the discovery learning model with ATI, and one control class receiving conventional learning. Data were collected using a test on trigonometry concept comprehension and a self-efficacy questionnaire. Data analysis was performed using inferential statistics, specifically an independent samples t-test with SPSS version 26.0. The results showed that the implementation of the discovery learning model with ATI significantly improved both students' mathematical concept comprehension and self-efficacy, with a significance level of $0.006 < 0.05$. However, the effect on self-efficacy was not statistically significant in isolation ($p = 0.089 > 0.05$), though a positive trend was observed. These findings suggest that the discovery learning model with ATI is effective in improving students' mathematical understanding and may contribute to self-efficacy development in mathematics learning.

Keywords: Aptitude Treatment Interaction; Discovery Learning; Mathematical; Self-Efficacy; Trigonometry.

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1. Introduction

Mathematics is a discipline that has a strategic role in the education system, especially in responding to the demands of 21st century learning. Mathematics learning not only aims to train numeracy skills, but also to form an analytical, logical, critical, and systematic mindset. Therefore, mathematics is designated as a compulsory subject at every level of education. This provision is in line with Article 37 of the Law of the Republic of Indonesia Number 20 of 2003 concerning the National Education System which emphasizes that mathematics must be taught in primary and secondary education. In addition, Government Regulation Number 4 of 2022 concerning National Education Standards states that graduate competency standards cover three domains, namely knowledge, skills, and attitudes. Thus, mathematics learning needs to develop these three aspects in a balanced manner in order to form hard skills and soft skills that are relevant to the needs of students (Ministry of Education and Culture, 2013).

In a cognitive context, understanding concepts is an important foundation so that students are able to learn mathematics meaningfully. Concepts allow students to relate to, explain, and apply mathematical ideas appropriately. NCTM emphasizes that mathematics learning at the secondary school level must be oriented towards the development of conceptual understanding. This is strengthened by Permendiknas Number 37 of 2018 which emphasizes that mathematics learning in secondary school is directed at the ability to understand concepts, explain the relationships between concepts, and apply algorithms accurately (De Zeeuw, Craig, & You, 2013). A number of studies have also shown that understanding concepts is the main basis for meaningful learning of mathematics (Kent & Foster, 2015). Students who have a good conceptual understanding tend to be able to represent concepts in various forms and use them in contextual problem solving.

However, these achievements are inseparable from affective factors, especially self-efficacy, namely an individual's confidence in his ability to complete academic tasks. The importance of understanding mathematical concepts is also reflected in the Programme for International Student Assessment (PISA) assessment which assesses the ability to understand concepts, reason, and solve problems in a real context (OECD, 2019). The PISA results show that Indonesia's math literacy score is still low, at 379 and is ranked 73rd, below the OECD country average of around 500. This condition shows the need to improve conceptual understanding, especially in the domains of Space and Shape and Change and Relationships, which are closely related to trigonometric material in understanding the relationship between angle and length of sides and their application in contextual situations.

Studies have revealed that students still face difficulties in understanding mathematical concepts, such as misconceptions and the tendency to learn procedurally without understanding the meaning of concepts (Ross & Wilson, 2012). As a result, learners tend to memorize formulas without mastering the essence of the concept, so that the process of assimilation and accommodation of knowledge does not take place optimally (Milligan & Wood, 2010). On the other hand, research shows that students with high self-efficacy are more diligent, confident, and able to face the challenges of learning mathematics (Schunk & DiBenedetto, 2014). The OECD report (2022) also states that low confidence correlates with low mathematical literacy. Therefore, strengthening conceptual understanding needs to be accompanied by efforts to increase self-efficacy through active learning and paying attention to differences in initial abilities.

Affective aspects that include beliefs, attitudes, and emotions also affect learning success (McLeod in Wadsworth et al., 2020). Self-efficacy, as stated by Bandura (1997), refers to an individual's confidence in his or her ability to complete tasks and achieve learning goals. Learners with high levels of self-confidence tend to be more persistent and brave in the face of challenges, while those with low self-efficacy tend to avoid difficult tasks (Santrock, 2009). The results of PISA 2018 also show that there is anxiety and low confidence in mathematics learning which has an impact on low self-efficacy (OECD, 2019).

One approach that has the potential to improve conceptual understanding is the discovery learning model based on constructivism theory, where learners actively discover and build knowledge through learning experiences (Uside et al., 2013). However, the application of this model has challenges, especially in classrooms with diverse initial abilities, which can affect the learning effectiveness and self-efficacy of learners (Cronbach & Snow, 1977). Therefore, a strategy is needed that is able to accommodate these differences in abilities.

The Aptitude Treatment Interaction (ATI) strategy offers a learning approach that adapts treatment to the characteristics and ability levels of the learner (Snow, 1989). Previous research has shown that the application of ATI can improve students' understanding of concepts, learning outcomes, and confidence (Inda & Widjayanti, 2018). Based on this description, it can be concluded that low conceptual understanding and self-efficacy are still challenges in mathematics learning, especially in trigonometry materials (Fitri, 2017). Therefore, this study is focused on the application of the discovery learning model combined with the Aptitude Treatment Interaction strategy as an effort to improve students' understanding of mathematical concepts and self-efficacy, so that learning becomes more effective and in accordance with their learning characteristics.

2. Literature Review

Mathematics is seen as a universal science that is the foundation of modern technological development and plays a strategic role in various disciplines and the development of human mindsets (Hill & Ball, 2004). Mathematics not only has a cognitive dimension, but also has social, cultural, and historical aspects. Reuben Hersh (1997) emphasized that mathematics is a product of culture that develops in response to the dynamics of society. In the context of education, mathematics learning is a process of educational interaction that aims to build a meaningful understanding of concepts, principles, and procedures, as well as develop thinking skills to solve real-life problems (OECD, 2019). The National Council of Teachers of Mathematics (NCTM, 2023) emphasizes that effective learning is oriented towards concept understanding, reasoning, communication, connections between concepts, and problem-solving, not just procedural memorization.

In line with constructivism, mathematics learning is active because students build knowledge through exploration and reflection based on previous experiences (Bruner, 2019). A student-centered approach is important in improving the quality of learning. In addition to the cognitive aspect, affective factors such as motivation and self-efficacy also determine the success of learning mathematics (Zimmerman & Schunk, 2000). Mathematics is taught at every level of education as a means of developing logical and adaptive thinking skills. Steinbring (2005) emphasized that learning mathematics as a constructive process gives room for the achievement of high-level cognitive goals. Various studies have also shown that learning mathematics is able to develop thinking skills and problem-solving strategies (Zhao et al., 2022).

Conceptually, Nisa & Arliani (2023) define mathematics as a logic-based science that includes geometry, analysis, and algebra. Johnson and Rising (1972) in Puspitasari (2022) view mathematics as a logical thinking pattern. Ernest (1994) states that mathematical knowledge is a priori, derived from the law of non-contradiction, and related to the context of justification. Bruner's theory emphasizes the importance of structure and concepts in learning (Suherman et al., 2003), while Carpenter and Lehrer (Romberg & Fenema, 2009) explain that the construction of mathematical knowledge involves the activity of connecting, applying, reflecting, and building new understandings. Mathematics learning is also influenced by two perspectives, namely as a collection of rules and as a structured logical system (Mononen, 2022).

The ability to understand mathematical concepts includes mastery of mathematical ideas, relationships, and principles. Nitko & Brookhart (2011) distinguish between concrete concepts and abstract concepts. Understanding is reflected in the ability to define, give examples, identify nonexamples, and relate concept components. Skemp (Kent & Foster, 2015) states that conceptual understanding is characterized by the ability to know the reasons behind the procedure. Kilpatrick, Swafford & Findell (2002) emphasizes that conceptual understanding includes interpretation, operation, and connections between concepts. Baroody et al. (2017) and Cheung et al. (2018) refer to understanding concepts as "knowing why". The National Research Council (2010) affirms that conceptual understanding allows learners to think generatively and choose the right solution strategies. Indicators of concept comprehension include example identification, visual representation, application of algorithms, verbal communication, and idea connections (Murizal et al., 2012).

Mathematical self-efficacy refers to an individual's belief in his or her ability to complete mathematical tasks (Dimopoulou, 2012). Bandura (1994) presents four sources of self-efficacy: verbal persuasion, vicarious experiences, mastery experiences, and affective factors. The dimensions of self-efficacy include level, generality, and strength. Research shows that self-efficacy affects motivation, effort, perseverance, and mathematical achievement (Pajares & Miller, 1994). Individuals with high self-efficacy tend to be more persistent in facing challenging tasks (Rahmi et al., 2017). Thus, self-efficacy is an important factor in the success of learning mathematics.

The Discovery Learning model places learners as active subjects who discover concepts through observation, data collection, analysis, and conclusion drawing (Bruner, 2019). This model is based on constructivism and has been proven to improve understanding of concepts and active learning (Uside et al., 2013). The stages include orientation, hypothesis formulation, data collection and analysis, verification, and generalization (Veermans, 2013). The advantages of discovery learning include increasing motivation, retention, and analytical thinking skills (Hanafiah, 2012).

The Aptitude Treatment Interaction (ATI) strategy introduced by Cronbach & Snow emphasizes the adaptation of learning behaviors to the learner's initial abilities (Cronbach & Snow, 2022). ATI views that the effectiveness of learning depends on the suitability between aptitude and treatment (Pamungkas & Afriansyah, 2017). This approach groups students based on their ability level and provides different treatment according to their individual characteristics (Serlina & Leonard, 2021). Research shows that ATI is effective in improving learning outcomes, participation, and self-efficacy (Burns et al., 2017).

The integration of Discovery Learning with the ATI strategy has a theoretical and empirical foundation in improving students' understanding of mathematical concepts and self-efficacy (Inda & Widjayanti, 2018). Based on this frame of thinking, the research hypothesis states that the application of ATI-based Discovery Learning has an effect on the understanding of mathematical concepts and self-efficacy of high school class X students, both simultaneously and partially.

3. Materials and Method

This study used a quasi-experiment design with a non-equivalent pretest–posttest control group pattern. Although it involved a control group, the design was not yet fully able to control external variables that could potentially affect the results. The treatment applied was in the form of the Discovery Learning model in two forms, namely Discovery Learning with the Aptitude Treatment Interaction (DL-ATI) strategy in the experimental class and Discovery Learning without the strategy in the control class. The study examined two bound variables, namely the ability to understand mathematical concepts and self-efficacy. The study was carried out at SMA Negeri 3 Tarakan in the even semester of the 2024/2025 school year with trigonometry material for eight meetings (including pretest and posttest). The study population was all students of grade X, with a randomly selected sample from eight classes to determine the experimental and control groups.

Data collection was carried out through tests and questionnaires. The test in the form of a description is used to measure the understanding of mathematical concepts through pretest and posttest, which are arranged based on a grid according to indicators, basic competencies, and lesson plans. The self-efficacy questionnaire uses a five-category Likert scale with 30 favorable and unfavorable statements (Ghozali, 2011).

Data analysis was carried out descriptively and inferentially. Descriptive statistics include average, variance, standard deviation, maximum–minimum scores, and percentage of completeness based on KKM 70. The assessment of concept understanding refers to Cai, Lane, and Jababsin (1996) with certain modifications, while the category of self-efficacy is determined based on the ideal average and the ideal deviation standard.

4. Results and Discussion

Research Results

This research was carried out at SMA Negeri 3 Tarakan in the even semester of the 2024/2025 school year for two weeks, namely July 15-25, 2024. The research population includes all students of class X consisting of four classes. Through the purposive sampling technique, class X-1 was selected as the experimental class and X-4 as the control class. The experimental class consisted of 30 students, while the control class consisted of 32 students. Learning in the experimental class lasted for three meetings with the application of the Discovery Learning model based on Aptitude Treatment Interaction (ATI).

The results of the descriptive analysis showed that the average score of the experimental class (82.57) was higher than that of the control class (73.94). The maximum score in both classes was 100, while the minimum score of the experimental class (60) was higher than that of the control class (52.5). Based on the school's assessment criteria, most of the students in the experimental class were in the very good and good category, and there were no students in the poor category. On the other hand, in the control class, five students were still found in the less category. In general, the category distribution shows that the experimental class has more even achievements at good and excellent levels.

The analysis prerequisite test showed that the data of the two classes were normally distributed based on the Kolmogorov–Smirnov test at the level of 0.05 with a significance value of 0.200 so that H_0 was accepted. The homogeneity test using the Homogeneity of Variance Test yielded a significance value of 0.643 (≥0.05), which means that the variance of the two classes is homogeneous. After meeting the assumptions of normality and homogeneity, an independent samples t-test is carried out. Self-efficacy analysis showed that all indicators (SE-1 to SE-7) had a relatively high average score, with a mean range of around 3.13–3.65. In the level dimensions (SE-1 and SE-2), students show strong confidence in facing obstacles and variations in the difficulty level of math tasks.

Discussion of research results

The discussion of the results of the study shows that the application of Discovery Learning with the ATI strategy has a positive influence on increasing the understanding of mathematical concepts compared to conventional learning. These findings are in line with the opinion of Suendarti & Liberna (2021) who stated that discovery-based learning encourages more meaningful understanding of concepts, especially in trigonometry materials. The principle of discovery learning, which emphasizes the active involvement of students in observing, discussing, and deducing learning concepts on their own, helps them understand the relationship between concepts, not just memorize formulas.

The aptitude treatment interaction strategy also strengthens the effectiveness of learning because the treatment is tailored to the ability of the learner. High-ability students get the opportunity to learn independently, while moderate- and low-ability students get more targeted guidance. This is in line with Amutawan et al. (2019) who stated that the ATI strategy is able to facilitate the gradual understanding of trigonometry concepts and minimize learning barriers. Discussion and presentation activities also deepen understanding through the process of re-revealing concepts that have been learned.

On the other hand, conventional learning that is more teacher-centered causes students to tend to be passive and less explore concepts in depth, so the understanding formed is relatively lower. However, this study shows that the application of ATI-based discovery learning has not had a significant effect on self-efficacy. This can be explained through the theory of self-efficacy which asserts that self-confidence develops through repeated experiences of success over a long period of time. Because the study lasted for a limited duration and focused more on the cognitive aspect, changes in self-efficacy have not been seen significantly.

Overall, these findings support the results of the research of Maulidia, Lisa Nur et al. (2025) which states that discovery learning based on aptitude treatment interaction is effective in improving the understanding of mathematical concepts in trigonometry materials. However, to optimize the increase in self-efficacy, additional strategies are needed that are oriented towards strengthening confidence and a continuous learning experience.

5. Conclusion

Based on the results of the analysis and discussion, it can be emphasized that the application of the Discovery Learning model combined with the Aptitude Treatment Interaction (ATI) strategy has a positive impact on the ability to understand mathematical concepts of class X students of SMA Negeri 3 Tarakan. Learning that is oriented towards active concept discovery and adjustment of treatment according to students' initial abilities is proven to help students understand trigonometry material better. Nonetheless, the increase in self-efficacy has not shown a significant effect partially. This indicates that the formation of self-efficacy is not only determined by the learning model, but is also influenced by continuous processes, learning experiences, and consistent environmental support.

Theoretically, these findings are in line with the view of constructivism which emphasizes that conceptual understanding is formed through the active involvement of learners in the process of discovering knowledge. Practically, the use of Discovery Learning with the ATI strategy can be an effective learning alternative in improving understanding of mathematical concepts, especially in trigonometry materials, while still considering the characteristics and initial abilities of students. Simultaneously, this model has a significant effect on the ability to understand mathematical concepts and self-efficacy, which shows that active involvement and learning adjustment are able to develop the cognitive and affective aspects of students. In particular, this model improves students' ability to understand concepts, connect between concepts, and apply concepts in problem solving. In addition, ATI's strategy of adjusting treatment based on initial ability also encourages confidence and perseverance in completing math tasks. Thus, the application of Discovery Learning combined with the ATI strategy can be declared effective in improving the understanding of mathematical concepts of class X students of SMA Negeri 3 Tarakan.

Based on these results, several suggestions can be proposed. Mathematics teachers are advised to utilize the Discovery Learning model with the ATI strategy as an alternative to learning, by paying attention to the differences in students' initial abilities so that the learning process takes place more effectively and is able to increase concept understanding and self-efficacy. Schools are expected to provide support through training or mentoring to improve teachers' competence in implementing the model. Students are also expected to play an active role in the learning process so that the understanding of concepts and learning outcomes can develop optimally. For future researchers, it is recommended to expand the scope of research on different subjects, materials, or educational levels, as well as examine other variables such as problem-solving skills, critical thinking, and learning motivation with a more diverse research design to obtain more comprehensive results.

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