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Application of Conditional Monte Carlo Simulation in Determining European Option Contract Pricing (Case Study on Toyota Motor Corporation (TM) Stock)

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Abstract*, When making investment decisions, it is crucial for investors to consider various risks that may arise, both in the short and long term. One method to measure risk is through volatility. Volatility represents a statistical measurement of the degree of price variation over a specific period, expressed as volatility (σ) (Aklimawati & Wahyudi, 2013). This study aims to discuss the pricing of European option contracts using Conditional Monte Carlo simulation and the Black-Scholes method. The data used in this study is secondary data obtained from Yahoo Finance. The data consists of quantitative information, namely the monthly closing prices of Toyota Motor Corporation (TM) stock, spanning 5 years from July 1, 2019, to July 1, 2024, yielding 60 data points. In this research, the pricing of European call option contracts was calculated using Conditional Monte Carlo simulation and the Black-Scholes method. The study concludes that European option contract pricing can be determined using two methods: Conditional Monte Carlo simulation and the Black-Scholes method. Conditional Monte Carlo simulation can be employed to calculate European option prices in a structured manner, utilizing stochastic volatility estimated through the Ordinary Least Squares (OLS) method. The two methods yield differing option prices; Conditional Monte Carlo simulation produces lower option price estimates with relatively lower error values compared to the Black-Scholes method at every strike price. The lower estimates from Conditional Monte Carlo simulation are due to its consideration of stochastic changes in volatility, whereas the Black-Scholes method results in higher prices due to its assumption of constant volatility. The comparison demonstrates that Conditional Monte Carlo simulation provides cheaper price estimates under market conditions with non-constant volatility, despite requiring higher computational time compared to the Black-Scholes method.*

Keywords*: Conditional Monte Carlo Simulation, Black-Scholes, Ordinary Least Squares (OLS)*

1. INTRODUCTION

Investment is now a crucial aspect of achieving long-term wealth growth and protection in today's dynamic and competitive financial world. Investment can be an asset that is intended to gain profit in the future by considering the time and risk involved in investing.(Megis and Arnellis, 2022).

The derivatives market is one type of investment that is increasingly popular because it offers a variety of financial instruments that allow market players to speculate, protect themselves from risk, and better manage their portfolios. Investors can take advantage of price fluctuations in various assets, such as stocks, commodities, and currencies through derivative contracts such as futures and options. Some examples of investments that are widely used by investors are gold investment, land investment, and stock investment. These three investments have different risks, especially stock investment because the risk is difficult to predict. So, more and more people are challenged to invest in stocks(Ratnasari et al., 2017).

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Stock options are agreements or contracts between buyers and sellers, in which the seller grants the buyer the right, but not the obligation, to buy or sell shares at a specified price and time (Brigham and Houston, 2011). If the option holder does not exercise his rights until the expiration date, the rights will be lost, so that the options held will no longer have value.(Syam et al., 2018)

Options can be used to maximize profits through greater leverage while minimizing risk. When making investment decisions, it is important for investors to consider the various risks that may arise, both short-term and long-term. One way to measure risk is through volatility. Volatility is a statistical measure of the degree of price variation from one period to the next, expressed as volatility (σ) (Aklimawati and Wahyudi, 2013). Higher volatility increases the likelihood of higher or lower underlying asset prices at maturity. Therefore, volatility values need to be interpreted because they fluctuate randomly and cannot be observed directly.(Dharmawan et al., 2011).

Option pricing, has several methods that have been developed to facilitate investors in making investment decisions regarding options. Some of the methods that are widely used include the Black-Scholes method, Monte Carlo simulation, and the Binomial method. The Black-Scholes method was developed by Fisher Black and Myron Scholes in 1973. The Black-Scholes method can only be used for European-type options that are valid at expiration and assume that the variance or volatility of the stock price is constant during the life of the option is known and also only applies to stocks that do not provide dividends. Various factors, including stock price, strike price, volatility, and constant interest rates, can have a significant impact on option prices(Tandelilin, 2010).

Monte Carlo simulation is a numerical approach that utilizes random simulations of stock prices and stock price fluctuations to evaluate and model complex phenomena. Option pricing, this method can be used to model the price movements of the underlying asset and generate probability distributions of future option prices. Monte Carlo simulation itself has the advantage of handling assets with complex variable characteristics, allowing the use of more realistic assumptions. However, the main shortcoming of Monte Carlo simulation is that it typically converges at a rate proportion to $m^{-\frac{1}{2}}$ where m is the number of samples in Monte Carlo simulation (Liang and Xu, 2020). Therefore, it is necessary to develop Monte Carlo simulation with variance reduction techniques to achieve convergence in Monte Carlo simulation development of Monte Carlo simulation. These variance reduction techniques include Antithetic Variations, Stratified Sampling, Control Variate, Conditional Monte Carlo (conditional Monte Carlo simulation).

Conditional Monte Carlo simulation is developed to estimate the conditional expectation value of a function by sampling from an unconditional distribution. Conditional Monte Carlo utilizes variance reduction techniques. Variance can be reduced by transforming variables using conditional expectation values. This will reduce the spread of the data and will be closer to the real value.(Astuti et al., 2021).

A study conducted by Astuti et al., (2021)studied option pricing using the Conditional Monte Carlo method and the Faure quasi-random sequence. The study concluded that standard Monte Carlo simulation and conditional Monte Carlo simulation using the Faure quasi-random sequence showed greater accuracy.

This study discusses the pricing of European option contracts using the conditional Monte Carlo simulation method and Black-Scholes. Therefore, the author raises the title "Application of Conditional Monte Carlo Simulation in Determining the Price of European Type Option Contracts (Case Study of Toyota Motor Corporation (TM) Stocks)"

2. RESEARCH METHODS

The data used in this study is secondary data obtained from https://yahoofinance.com/. The data is quantitative data, namely the monthly closing price of Toyota Motor Corporation (TM) shares over a period of 5 years, starting from July 1, 2019 - July 1, 2024 has 60 outputs.

In this study, the calculation of the price of a European call option contract was carried out using conditional Monte Carlo simulation and the Black-Scholes method. The applications that will be used are Microsoft Excel 2013 and Phyton. The following are the stages that will be carried out in this study.

1. Collecting historical data of monthly closing prices of Toyota Motor Corporation (TM) shares from July 1, 2019 – July 1, 2024.
2. Determine the rate of return from historical data on stock closing prices using equation (2.10).
3. Determine the factors that influence the option price such as the stock price (S_0), strike price (K), expiration date (T), and risk-free interest rate (r).
4. Determining the price of a European-type call option using conditional Monte Carlo simulation, as follows:

a) Calculating stochastic volatility using equation (2.16) by generating random numbers with a standard normal distribution. The estimation parameters to be used to calculate stochastic volatility are determined by equation (2.17).
b) Calculate volatility by averaging the squared volatilities in step a) which will be used in calculating the option price.
c) Generating random numbers in the form of standard normal distributed random numbers.
d) Calculate the estimated value of shares in a specified period using equation (2.9).
e) Calculate the price of a European type call option using equation (2.20).
f) Calculate the average option price and get the European type option price.
g) Repeat steps (a) to (f) 100, 1000, 2000, 3000, 4000, and 5000 simulations.

3. RESULTS AND DISCUSSION

This study uses historical data of monthly closing prices of Toyota Motor Corporation (TM) shares. The monthly closing price data is data for five years starting from July 1, 2019 to July 1, 2024 with 60 outputs, which are obtained fromwww.yahoo.finance.com. Rate of return. Calculations are made using Microsoft Excel and Python software.

Calculating Return and Expected Return

Price put and call options with conditional Monte Carlo simulation require return values, volatility, determining factors that affect option prices which in this calculation are calculated using Microsoft Excel. The stock return value is calculated based on stock closing price data using equation (2.10). First, the return value of the monthly stock closing price for the five-year research period will be calculated, namely Toyota Motor Corporation (TM) shares with a period from July 1, 2019 to July 1, 2024 which has 60 outputs. For example, the calculation of the 1st Toyota Motor Corporation (TM) stock return on July 1, 2019 and the closing stock price on August 1, 2019 so that the TM return value on July 1, 2019 is$(P_{TM.JK2} = 129.05)(P_{TM.JK1} = 130.63)$

$$R_{TM1} = \ln\left(\frac{P_{TM.JK1}}{P_{TM.JK2}}\right)$$

$$= \ln\left(\frac{130.63}{129.05}\right)$$

$$= -0.012168988$$

Next, for the stock returns to 2, 3, …, 60 TM are calculated in the same way, the complete results of the return calculation are presented in Appendix 2. After obtaining the stock return, the next step is to determine the expected stock return value. Expected return is calculated based on the return value using equation (2.11), the calculated expected stock return value of TM is:

$$\overline{R} = \frac{1}{n}\sum_{t=1}^{n} R_t$$

$$\overline{R}_{TM} = \frac{1}{60}\sum_{t=1}^{503} R_{TM.JK}$$

$$= \frac{1}{60} \times (R_{TM1} + R_{TM2} + \cdots + R_{TM60})$$

$$= \frac{(0.01217 + 0.02912 + \ldots + (-0.25))}{60}$$

$$= -0.00354$$

So, it is obtainedexpected returnTM shares amounting to-0.00354.

Determining Factors Affecting Option Prices

The following are factors that influence option prices:

1. **Initial Share Price (S_{t_0})**

   The initial stock price to be used is the closing stock price on July 1, 2019 of $129.05.

2. **Agreement Price (K)**

   The strike prices that will be used are $125, $130, and $135.

3. **Due Date (T)**

   The expiration time of the option contract is1year.

4. **Risk Free Interest Rate (r)**

   The interest rate is assumed to be constant at 6% per year according to the interest rate from Bank Indonesia.

Determining Call Option Pricing with Monte Carlo Simulation

In calculating option prices with conditional Monte Carlo simulations, stochastic volatility will be used, with volatility not considered constant, but changing over time. Therefore, the first step is to simulate the stochastic volatility trajectory, then use the results to calculate the stock price at maturity. By knowing the volatility trajectory, the average value of its square will be used to calculate the expected payoff of the option more accurately.

1. Calculating Stochastic Volatility(V_t)

Stochastic volatility represents the level of volatility that changes stochastically or randomly over time. It can be calculated using equation (2.12), namely:(V_t)

$$V(R_t) = (R_t - \overline{R})^2$$

$$V_{tTM1} = (-0.01216 - (-0.00345))^2$$

$$= 0.0000744$$

Next, for the stochastic volatility of TM.JK shares in months 2, 3, ..., 60 is calculated in the same way. So that the results of the Stochastic Volatility calculation.(V_t)

2. Calculating Stochastic Volatility Changes(dV_t)

In this calculation of stochastic volatility changes, the stochastic volatility changes will be calculated at time. Thus, we get:$(V_{t+1} - V_t)$

$$dV_{tTM.JK1} = 0.000654 - 0.0000744$$

Next, for the change in the stochastic volatility of TM.JK shares in months 2, 3, ..., 60, it is calculated in the same way. So, the calculation resultsStochastic Volatility Change(dV_t).

3. Parameter Estimation

The process of simulating the stock price trajectory using the mean-reverting model requires estimating three main parameters, namely α $(alpha)$, κ $(kappa)$, and σ $(sigma)$. which are assumed to remain constant during the estimation period. The process of simulating the stock price trajectory using the mean-reverting model requires estimating three main parameters, namely α $(alpha)$, κ $(kappa)$, and σ $(sigma)$. The assumption used is that these parameters remain constant during the estimation period. The mean-reverting process equation is expressed by equation (2.16), namely:

$$dV_t = \alpha(\kappa - V_t) + \sigma V_t dW_{t,1}$$

$$V_{t+1} - V_t = \alpha \times \kappa - \alpha \times V_t + \sigma V_t dW_{t,1}$$

Equation (2.17), namely:

$$Y_t = a + bX_t + e_t$$

is a linear regression model used to estimate the parameters of the mean-reverting process of stochastic volatility in equation (2.16). With is the change in asset price over time. $Y_t = dV_t tt + 1$And. $X_t = V_t$The coefficients are, while are the coefficients, and are the residuals that describe the random component. The parameter value estimates for the stochastic volatility model obtained with the help of Excel software are shown in Table 1.$a\alpha \times \kappa b - \alpha e_t$

Table1. Parameter values α, κ, dan σ

Parameter	Mark
α	0.988108724
κ	−0.005430602
σ	0.010166175

4. Simulating Stochastic Volatility

Simulating the volatility trajectory with simulation is done by generating random numbers that reflect various possible historical volatility movements. Simulating the volatility trajectory will use the mean-reverting model with the help of Python, using equation (2.16) to calculate volatility at each time period step. Simulation results from stochastic volatility datashows 100 simulations of volatility trajectories following a mean-reverting model with a consistent increase over time. The variation across trajectories is small, indicating that the parameters in the mean-reverting model and the initial volatility used produce a stable pattern across each simulation trajectory, showing an increase in volatility under the prevailing conditions.

5. Calculating the Average Squared Volatility (σ^{-2})

The value provides information about the volatility level trend over a certain period, including the magnitude of extreme price changes. This process helps to understand volatility changes and their impact on the price of financial instruments. This process will be assisted by Python software which begins by collecting volatility data at various points in time throughout the analysis period. Each volatility value is then averaged and squared, so that the average of the squared volatility is obtained ($\overline{\sigma}^2$) of 0.00066.

6. Calculating Call Option Prices with Conditional Monte Carlo Simulation

The calculation of the price of a European-type call option using the conditional Monte Carlo method begins with generating random numbers with a standard normal distribution. Then calculate the value S_t in each period using equation (2.9). After that, calculating the price of the call option can be calculated with the help of Phyton, with the value of the call option according to the conditional expectation with equation (2.20) for the call option. The simulation process was carried out with agreed prices of 125, 130, and 135. The calculation results obtained tend to differ, because the simulation process uses random numbers.

a) Strike price 125

The calculation of the price of this call option is carried out with a strike price of 125 using a conditional Monte Carlo simulation repeated 100 times.(K)1000, 2000, 3000, 4000 and 5000times. Results generated from each simulation are then averaged to obtain call option price estimate. The call option prices can be seen in Table 2.

Table 2. Simulated Call Option Price Calculation Data

Conditional Monte Carlo with K = 125 (Price in Dollars).

Number of Simulations	Call Option Price ($)	ErrorRelative Buy Option
100	12.1604	0.3270
1000	13.7927	0.2718
2000	18.2786	0.0184
3000	18.2603	0.0148
4000	18.2403	0.0127
5000	18.2074	0.01170

Table 2 shows the results of calculating the call option price using conditional Monte Carlo simulation with a strike price of 125. The call option price increases and becomes more stable as the number of simulations increases, with a convergence value of around $18.2074 in 5000 simulations. Meanwhile, the relative error reflecting the degree of deviation of the estimate from the true value continues to decrease, from 0.3270 in 100 simulations to 0.0117 in 5000 simulations. This shows that increasing the number of simulations not only improves the stability but also the accuracy of the calculation results, so that Monte Carlo simulations with a larger number of simulations provide more optimal option price estimates.

b) Strike price 130

The calculation of the price of this call option is carried out with a strike price of 130 using a conditional Monte Carlo simulation repeated 100 times.(K)1000, 2000, 3000, 4000 and 5000time.The call option prices can be seen in Table 3.

Table2. Simulated Call Option Price Calculation Data

Conditional Monte Carlo with K = 130 (Price in Dollars).

Number of Simulations	Call Option ($)	ErrorRelative Buy Option
100	14.2969	0.1090
1000	14.6291	0.1303
2000	15.2910	0.0153

3000	15.2541	0.0173
4000	15.2156	0.0150
5000	15.2145	0.0133

Table 3 shows the results of calculating the call option price using conditional Monte Carlo simulation with a strike price of 130. The call option price tends to increase and become more stable with the increase in the number of simulations, with a convergence value of around $15.2145 in 5000 simulations. At the same time, the relative error decreases significantly, from 0.1090 in 100 simulations to 0.0133 in 5000 simulations, reflecting a higher level of estimation accuracy. This shows that the more simulations used, the better the calculation results are in approaching the actual value, thus providing a more accurate call option price estimate.

c) Strike price 135

The calculation of the price of this call option is carried out with a strike price of 135 using a conditional Monte Carlo simulation repeated 100 times.(K)1000, 2000, 3000, 4000 and 5000time.The call option prices can be seen in Table 4.

Table 4. Simulated Call Option Price Calculation Data

Conditional Monte Carlo with K = 135 (Price in Dollars).

Number of Simulations	Call Option ($)	ErrorRelative Buy Option
100	9.7012	0.2066
1000	10.8340	0.1360
2000	11.1841	0.0268
3000	11.1959	0.0216
4000	11.0781	0.0184
5000	11.0155	0.0166

Table 4 shows the results of calculating the call option price using conditional Monte Carlo simulation with a strike price of 135. The call option price increases and approaches a stable value of around $11.0155 when the number of simulations is increased to 5000. On the other hand, the relative error which describes the level of inaccuracy of the estimation results shows a significant downward trend, from 0.2066 at 100 simulations to 0.0166 at 5000 simulations. These results indicate that the more simulations used, the more accurate the call option price estimate produced, so that the conditional Monte Carlo simulation method provides more stable and reliable results at a larger number of iterations.

Determining the Price of a Call Option Using the Black - Scholes Method

In calculating the price of an option contract using the Black-Scholes method, volatility is assumed to be constant throughout the time until maturity. This method assumes that the movement of stock prices follows a process determined by the risk-free interest rate, time to maturity, and constant volatility. With this approach, we can calculate the price of an option analytically using the Black-Scholes formula, with volatility still playing an important role in measuring the risk and fluctuation of stock prices during the option period.

1. Determining Volatility

The first step, the variance value of Toyota Motor Corporation (TM) shares will be calculated first using equation (2.10) on the return data. By using the help of Microsoft Excel 2013 software, the variance value of Toyota Motor Corporation (TM) shares is obtained as 0.005328. Based on the variance value of Toyota Motor Corporation (TM) shares obtained, the annual volatility value of Toyota Motor Corporation (TM) shares will be calculated using equation (2.11).

$$\begin{aligned}\sigma &= \sqrt{\mathrm{Var}(R_t) \times k} \\ &= \sqrt{0.005328 \times 12} \\ &= 0.252866\end{aligned}$$

Thus, the stock volatility value is 0.252866.

2. Calculating the Price of a Call Option Using the Black - Scholes Method

The calculation of the price of a European type call option using the Black-Scholes method is based on equation (2.14) as follows:

Call option price:

$$C_{(S_t,t)} = S_0 N(d_1) - K e^{-rT} N(d_2)$$

with

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T)}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln\left(\frac{129.05}{125}\right) + \left(0.06 + \frac{1}{2}(0.252866)^2\right)(1)}{0.252866\sqrt{1}}$$

$$d_1 = 0.395598$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_2 = 0.356378 - 0.252866\sqrt{1}$$

$$d_2 = 0.142732$$

The obtained price of a European type call option is:

$$C_{(S_t,t)} = 129.05N(0.395598) - 125e^{-0,06(1)}N(0.142732)$$

$$C_{(S_t,t)} = 18.88$$

Thus, the call option price is $18.88.In the same way for the call option price which will be calculated with the help of Phyton software, the call option prices obtained for different agreement values are completely presented in Table 5.

Table 3. Calculation Data for Call Option Price with

Black - Scholes Method (Price in Dollars).

Strike Price	Call Option ($)
125	18.88
130	16.23
135	13.87

Table 5 shows the results of calculating the call option price using the Black-Scholes method for several strike prices. One of the main assumptions of the Black-Scholes method is that volatility is considered constant during the option's validity period.

Comparing Option Prices Obtained by Conditional Monte Carlo Simulation and the Black Scholes Method

The calculation of the call option price using conditional Monte Carlo simulation and the Black Scholes method is obtained as in Table 6.

Table 4. Option Pricing with Monte Carlo Simulation

Conditional and Black Scholes Method

Strike price	Call Option ($) with conditional Monte Carlo simulation	Call Option ($) with Black Scholes method
125	18.2074	18.88

130	15.2145	16.23
135	11.0155	13.87

Table 6 compares the results of calculating the call option price using conditional Monte Carlo simulation with the Black-Scholes method at various strike prices. The results show that the call option price calculated using the conditional Monte Carlo simulation method is always lower than the Black-Scholes method.

4. CONCLUSION

Based on the research results, it can be concluded that the calculation of the European type option contract price is calculated using two methods, namely conditional Monte Carlo simulation and the Black-Scholes method. Conditional Monte Carlo simulation can be used to calculate the price of a European type option with structured steps, using stochastic volatility estimated using the Ordinary Least Squares (OLS) method. Conditional Monte Carlo simulation and the Black-Scholes method produce call option prices that tend to differ, conditional Monte Carlo simulation provides a lower option price estimate with a low relative error value compared to the Black-Scholes method at each strike price. Conditional Monte Carlo simulation is lower because it considers changes in volatility stochastically. While the Black-Scholes method provides a higher price because it uses the assumption of fixed volatility. The comparison results show that conditional Monte Carlo simulation is able to provide a cheaper price estimate in market conditions with non-constant volatility, although it requires more computing time than the Black-Scholes method.

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