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*Review Article*

# Determining the Price of Asian Type Call Option Contracts Using the Monte Carlo Stratified Sampling Method

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**Abstract:** Determining the price of option contracts is a crucial aspect of financial markets, particularly for investors aiming to manage risk and make informed investment decisions. In this study, the price of an Asian call option is calculated using the Monte Carlo Stratified Sampling method based on the stock price data of Tesla, Inc. (TSLA) from January 2021 to December 2023. This method has been proven to reduce variance compared to the Standard Monte Carlo simulation, leading to faster price convergence and more efficient results. The parameters used in the simulation include the initial stock price (S_0), number of simulations (N), maturity time (T)dividend = 0, risk-free rate (r), strike price ( K), and volatility $\sigma$ , which varies across different strata. The simulation results show that Monte Carlo Stratified Sampling produces a lower standard error and a more stable option price than the Standard Monte Carlo method.

**Keywords:** Asian Call Option; Standard Monte Carlo Simulation; Stratified Sampling Method

## 1. Introduction

Investment is the allocation of funds with the aim of obtaining profits in the future (Tandelilin, 2008). Options as instrument derivative the more popular Because its flexibility in manage risk volatile market . Options allows investors to limit potential loss without must sell asset basic , so that still can to obtain profit when condition market get better.

Option is contract that provides right to the holder For buy or sell a asset on price And the time that has passed determined ( Bodie , 2006). There is two type main options , namely option buy (call option) and option sell (put option). Based on its characteristics , options shared become option standard (Vanilla) traded on the stock exchange And non-standard (Exotic) options that are more complex . One of the type Exotic options are option path dependent , such as Asian options , which determine the payment based on average price asset during period contract For reduce volatility price extreme.

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In determine mark contract Asian options , used various methods , including the Black-Scholes , Binomial Tree , Trinomial Tree , and Monte Carlo models . Monte Carlo methods are flexible in handle distribution complex pricing , but own variation high yield If amount simulation limited ( Glasserman , 2003). By Because that , used Monte Carlo Variance Reduction techniques , such as stratified sampling , which divides population become a homogeneous stratum use increase accuracy estimat .

Study This apply method Stratified Sampling in determine price contract option buy Asia on price data closing Tesla, Inc. (TSLA) shares In addition that , research this also compares level convergence price contract calculated options using Standard Monte Carlo with Monte Carlo stratified sampling.

## 2. Research Methods

Study This using price data closing Tesla, Inc. (TSLA) shares from period January 2021 to December 2023 obtained from the site yahoo.finance.com. Study This use Standard Monte Carlo method and Monte Carlo Stratified Sampling with help Microsoft Excel and Python applications in determine price contract option buy Asian type.

### 2.1. Standard Monte Carlo Simulation

a. Data Collection and Parameters: Historical data TSLA shares were collected , then set parameters such as price share beginning ($S_0$ ), exercise price , risk (K)- free interest rate (r), amount simulation (N), maturity time (T)and volatility value ($\sigma$). Volatility share counted with :

$$\sigma=\sqrt{(1/(n-1)\sum_{t=1}^{n}(R_t-(R_t)^{-})^2}$$

Where $R_t$is the daily return and $(R_t)^{-}$ is the average return.

b. Simulation Movement Price Stocks : Price share simulated with using the Geometric Brownian Motion model :

$$S_t=[\![S]\!]_0\times\exp[(r-\sigma^2/2)\Delta t+\sigma Z_i\sqrt{\Delta t}]$$

Where number random in the form of number normally distributed .

c. Option Pricing: After obtaining the average stock price, the price of an Asian-type call option is calculated by:

$$C=e^{(-rT)}\max(S^{-}-K,0)$$

### 2.2 Monte Carlo Stratified Sampling Simulation

a. Division : Volatility share shared to in three strata: low , medium , and height . Division This aiming reduce Variance with grouping homogeneous data .

b. Determination Proportion And Sample : Proportion each stratum is counted And amount sample from each stratum is determined use formula Slovin :

$$n=M/(1+M([\![e]\!])^2)$$

where Mstates the population size and estates the level of error or mistake accepted.

c. Stock Price Movement Simulation per Strata: Each stratum, stock prices are simulated using the same Geometric Brownian Motion model as Standard Monte Carlo, but with different volatility values in each stratum.

d. Strata Results Combination: Option prices are calculated for each stratum, then the results are combined according to the stratum proportions:

C_total=∑_(i=1)^k▒〖p_i C_i 〗

Where p_i is the proportion of strataC_i And is price option for the istratum -

## 3. Results And Discussion

Implementation stratified sampling on price data closing Tesla, Inc. (TSLA) stock is done based on daily volatility of the stock price with the following steps:

a. Defining the Population: Daily volatility data from 753 TSLA stock closing prices is used as the population.

b. Determining Strata: Population shared become three strata based on volatility : low , medium , and tall each with 251 elements .

c. Count Strata Proportion : Proportion each stratum is counted as :

$$p = \frac{251}{753} = 0.333$$

d. Determining the Sample Size: Using the Slovin formula:

$$n = \frac{M}{1 + M(e)^2} = \frac{753}{1 + 753(0.05)^2} \approx 261$$

e. Determine Amount Sample Each Strata: Number sample per stratum:

$$n_i = p \times n = 0.333 \times 261 = 87$$

f. Calculating the Population Volatility Mean Estimate: The mean of each stratum is calculated and combined:

$$\bar{x}_n = \sum_{i=1}^{k} p_i \, \bar{x}_{n_i} = 0.0329$$

g. Calculating Population Variance Estimates: The variance of each stratum is calculated and used to estimate the population variance:

$$\widehat{var}(\mathrm{X}) = \sum_{i=1}^{k} p_i^2 \frac{var(x_{n_i} | n_i \in S_i)}{n_i} = \ 0.000000127$$

h. Count Hose Trust : With level 95% $t_{\alpha,n-1} = 1{,}96$confidence ( ):

$$0.0329 \pm 1{,}96\sqrt{0.000000127} = (0.03359, 0.0322015)$$

Results This show that the average volatility actual population is 0.0332 in hose trust said , shows representation good sample . The sample taken , then used For do simulation Monte Carlo Stratified Sampling in determine price contract option buy Asian type .

Simulation Monte Carlo Standard and Monte Carlo Stratified Sampling done so that produce difference mark price option as following :

Following is results comparison second method :

**Table 1.** Standard Monte Carlo Comparison and Monte Carlo Stratified Sampling

| N simulasi | Monte Carlo Standar | | Monte Carlo *Stratified Sampling* | |
|---|---|---|---|---|
| | Harga Opsi | Standart Error | Harga Opsi | Standart Error |
| 1.000 | 31.779014 | 1.728156 | 28.136768 | 1.527254 |
| 2.000 | 29.509836 | 1.106713 | 29.807436 | 1.118965 |
| 5.000 | 30.759295 | 0.742740 | 27.890202 | 0.657424 |
| 10.000 | 31.126002 | 0.536903 | 27.425862 | 0.474379 |
| 20.000 | 30.675063 | 0.373805 | 27.923363 | 0.333839 |
| 50.000 | 30.553562 | 0.237476 | 27.345674 | 0.210285 |
| 100.000 | 30.559786 | 0.167219 | 27.488463 | 0.148670 |
| 200.000 | 30.747085 | 0.119011 | 27.353165 | 0.105754 |
| 500.000 | 30.744732 | 0.075100 | 27.559403 | 0.067022 |
| 700.000 | 30.803241 | 0.063448 | 27.418595 | 0.056382 |
| 2.000.000 | 30.776138 | 0.037489 | 27.488752 | 0.033356 |
| 3.000.000 | 30.757791 | 0.036567 | 27.478240 | 0.022280 |
| 4.000.000 | 30.796535 | 0.030664 | 27.476024 | 0.022172 |
| **5.000.000** | 30.826571 | 0.023788 | 27.475256 | 0.022153 |
| 6.000.000 | 30.826547 | 0.025701 | 27.474858 | 0.022147 |
| **7.000.000** | 30.826323 | 0.023690 | 27.474530 | 0.022132 |

Results simulation show that Standard Monte Carlo Simulation stop on 7,000,000th simulation because standard error has approach zero , that is of 0.023690, and price option considered has reach convergence at $30.826323. While Monte Carlo Stratified Sampling simulation , price option from simulation stopped on 5,000,000th simulation because standard error has approach zero , which is 0.022153, and price option considered convergent at $27.475256.

This matter show that Monte Carlo Stratified Sampling more fast reach convergence with standard error more small .

## 4. Conclusion And Suggestions

Based on results research , can concluded that Stratified Sampling can used For share population that is volatility into the several strata. Taking sample in a way random done from every start as representative so that Variance from previous data can reduced . Sample random those that are used in Monte Carlo Stratified Sampling Simulation For determine price option buy Asian type .

Results Monte Carlo Stratified Sampling Simulation produce price more options fast reach convergence with a small standard error compared to with Standard Monte Carlo , so that can confirmed that method Stratified Sampling can increase accuracy estimate price option compared to Standard Monte Carlo .

Study Next , it is recommended For consider factor other like dividend And cost transaction so that can more increase accuracy estimate price option

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